Mathe-Grundlagen

1. Totale Ableitung von Fkt mit mehreren Variablen

Beispiel: $f(x,y) = x^2 y \qquad x = t^3 \qquad y = t+5$

$$f(x(t),y(t)) = t^6(t+5) = t^7 + 5t^6 \qquad \frac{dx}{dt} = 3t^2 \qquad \frac{dy}{dt} = 1$$

$$\boxed{\frac{df}{dt} = \underline{7t^6 + 30t^5} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}}$$

$$= 2xy \cdot 3t^2 + x^2 \cdot 1$$

$$= 2t^3(t+5) \cdot 3t^2 + t^6$$

$$= \underline{6t^6 + 30t^5 + t^6}$$

Aufgabe: $f(x,y) = x^3 y \qquad x = t^2 \qquad y = t^3 + 2t$

$$\frac{df}{dt} = \dots$$

Ableitungs-Schreibweisen

$$\dot{s} = s'(t) = \frac{ds}{dt}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt} \quad \Big|\cdot dt$$

2. $$df = \frac{\partial f}{\partial x}\cdot dx + \frac{\partial f}{\partial y}\cdot dy$$

$$f_2 - f_1 = \int_{f_1}^{f_2} df = \left[f \right]_{f_1}^{f_2}$$

3. $$\frac{d\ln(x(t))}{dt} = \frac{d\ln(x)}{dx}\cdot\frac{dx}{dt}$$

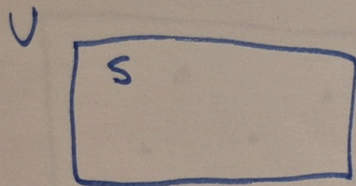
$$= \frac{1}{x}\cdot\frac{dx}{dt} \quad \Big|\cdot dt$$

$$d\ln(x) = \frac{dx}{x}$$

$$db$$

$\ln(x) =: b$

Grundbegriffe der Thermodynamik



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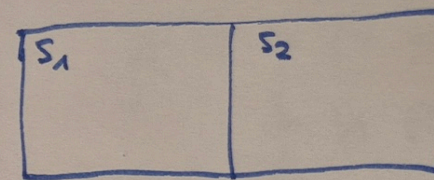


1. Hauptsatz der TD

$$dU = \delta Q - \delta W$$

innere Energie — Wärme — Arbeit

$$dU = T \cdot dS - p \cdot dV$$

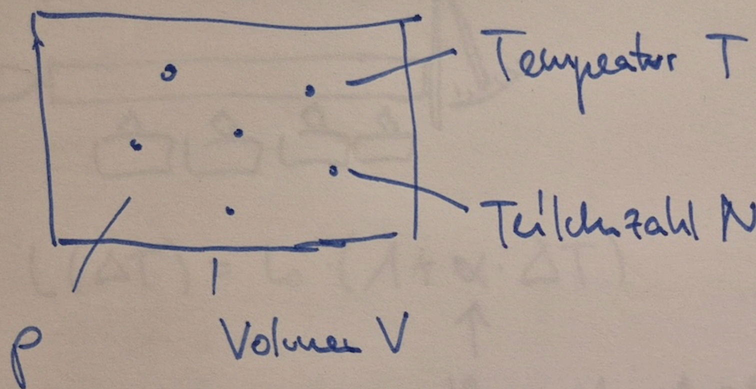
Temperatur — Entropie — Druck — Volumen

thermische Energie — mechanische Energie

Extensive Größe	Intensive Größe
V	p
S	T

Zustandsgrößen

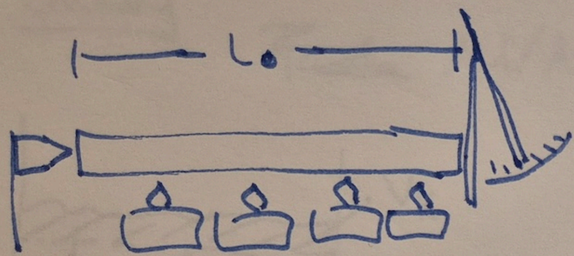
Idealer Gas



$$pV = N k_B T = n R T = m R_s T$$

$$n = \frac{N}{N_A} = \frac{m}{M}$$

Thermische Ausdehnung



$$L(\Delta T) = L_0 \cdot (1 + \alpha \cdot \Delta T)$$

lineare thermischer Ausdehnungskoeffizient

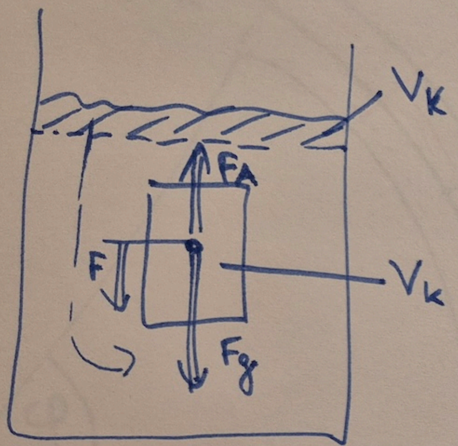
$$V(\Delta T) \approx V_0(1 + 3 \cdot \alpha \Delta T)$$

$$V(\Delta T) = V_0(1 + \gamma \cdot \Delta T)$$

kubisch thermische Ausdehnungskoeffizient

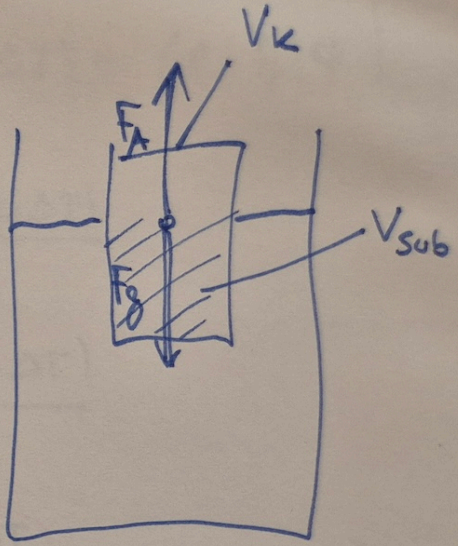
Erinnerung

~~Tauche~~ Auf/Abtrieb

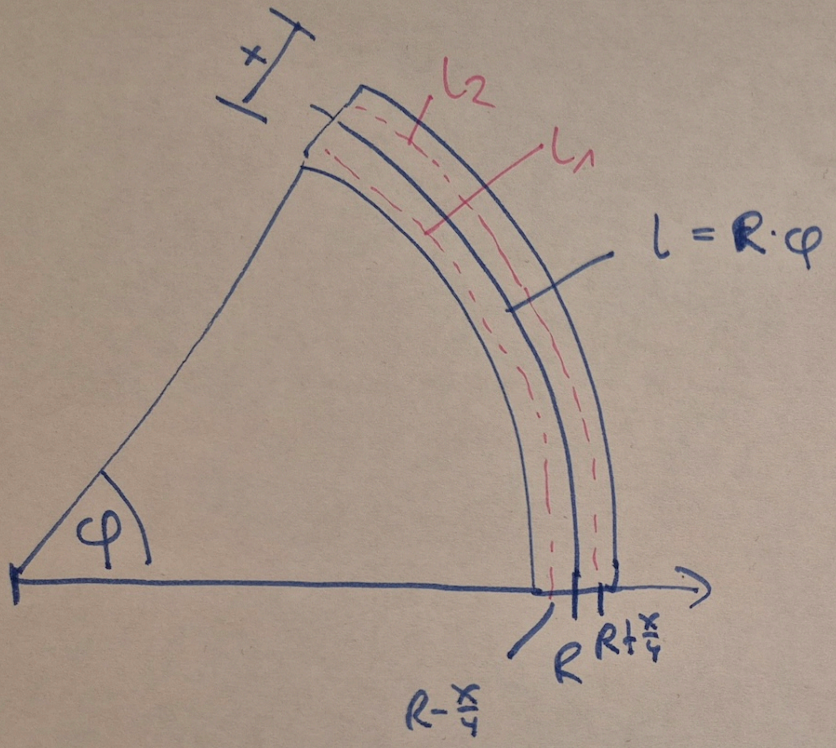


$$F = F_g - F_A$$
$$= m_K g - m_W \cdot g$$
$$= \rho_K V_K \cdot g - \rho_W \cdot V_K g$$
$$= (\rho_K - \rho_W) V_K \cdot g$$
$$= \left(1 - \frac{\rho_W}{\rho_K}\right) \cdot \rho_K \cdot V_K \cdot g$$
$$= \left(1 - \frac{\rho_W}{\rho_K}\right) \cdot m_K \cdot g$$

Schwimmen



$$F_g = F_A$$
$$m_K g = m_W g$$
$$\rho_K \cdot V_K \cdot g = \rho_W \cdot V_{sub} g$$
$$\frac{V_K}{V_{sub}} = \frac{\rho_W}{\rho_K}$$



$$\left| \begin{aligned} L_1 &= L_0(1+\alpha_1 \Delta T) = \left(R-\frac{x}{4}\right)\varphi \\ L_2 &= L_0(1+\alpha_2 \Delta T) = \left(R+\frac{x}{4}\right)\varphi \end{aligned} \right|$$

$$\varphi = \frac{L_0(1+\alpha_1 \Delta T)}{R-\frac{x}{4}}$$

$$\varphi = \frac{L_0(1+\alpha_2 \Delta T)}{R+\frac{x}{4}}$$

$$R = \frac{x}{4}\,\frac{2+(\alpha_1+\alpha_2)\Delta T}{(\alpha_2-\alpha_1)}$$

$\boxed{\text{s.goerig@gg.bildung-rp.de}}$

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